An Interesting Proof by Induction of Derivation

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Problem: *Use induction and the product rule to prove that if $f(x)$ is a differntiable function, then*

$$(f(x)^m)' = mf(x)^{m-1}f'(x),$$

for all integers $m \geq 1$.

Proof.

Consider the case where $m = 1$. Then $(f(x)^1)' = 1f(x)^{1-1}f'(x) \rightarrow f'(x) = f'(x)$.

Now, assume $(f(x)^m)' = mf(x)^{m-1}f'(x)$ for some integer $m > 1$. Then the left hand side of the equation is $(f(x)^{m+1})' = (f(x)^m f(x))' = (f(x)^m)'f(x) + f(x)^m f'(x) = [mf(x)^{m-1}f'(x)]f(x) + f(x)^m f'(x) = mf(x)^m f'(x) + f(x)^m f'(x) = (m+1)f(x)^m f'(x)$ for $m+1$. Similarly, the right hand side of the equation is $(m+1)f(x)^{(m+1)-1}f'(x) = (m+1)f(x)^m f'(x)$ for $m+1$.

Therefore, if $f(x)$ is a differentiable function then $(f(x)^m)' = mf(x)^{m-1}f'(x)$ for all integers $m \geq 1$.

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